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Out-of-equilibrium quantum field dynamics in external fields

F.J. Cao[a]

Departamento Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Avenida Complutense s/n, 28040 Madrid, Spain and
LERMA, Observatoire de Paris, Laboratoire Associé au CNRS UMR 8112, 61, Avenue de l'Observatoire, 75014 Paris, France

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Abstract. The quantum dynamics of the symmetry-broken $\lambda(\Phi^2)^2$ scalar-field theory in the presence of an homogeneous external field is investigated in the large-N limit. We consider an initial thermal state of temperature T for a constant external field $\mathcal{J}$. A subsequent sign flip of the external field, $\mathcal{J} \to -\mathcal{J}$, gives rise to an out-of-equilibrium nonperturbative quantum field dynamics. We review here the dynamics for the symmetry-broken $\lambda(\Phi^2)^2$ scalar N component field theory in the large-N limit, with particular stress in the comparison between the results when the initial temperature is zero and when it is finite. The presence of a finite temperature modifies the dynamical effective potential for the expectation value, and also makes that the transition between the two regimes of the early dynamics occurs for lower values of the external field. The two regimes are characterized by the presence or absence of a temporal trapping close to the metastable equilibrium position of the potential. In the cases when the trapping occurs it is shorter for larger initial temperatures.

PACS. 11.10.Wx Finite-temperature field theory – 11.15.Pg Expansions for large numbers of components (*e.g.*, $1/N_c$ expansions) – 11.30.Qc Spontaneous and radiative symmetry breaking

1 Introduction

Several important physical systems, as the ultrarelativistic heavy-ion collisions [1] and the early universe [2], present out-of-equilibrium dense concentrations of particles. The presence of these concentrations imply the need of out-of-equilibrium nonperturbative quantum field theory methods, as the large-N limit.

2 The model

We consider N scalar fields, $\boldsymbol{\Phi}$, with a $\lambda(\boldsymbol{\Phi}^2)^2$ self-interaction in the presence of an external field $\mathcal{J}$. The action and the Lagrangian density are given by

$$S = \int \mathrm{d}^4x \mathcal{L}\,, \tag{1}$$

$$\mathcal{L} = \frac{1}{2}[\partial_\mu \boldsymbol{\Phi}(x)]^2 - \frac{1}{2}m^2\boldsymbol{\Phi}^2 - \frac{\lambda}{8N}(\boldsymbol{\Phi}^2)^2 - \frac{m^4 N}{2\lambda} + \mathcal{J}\boldsymbol{\Phi}. \tag{2}$$

We restrict ourselves to the case where the symmetry is spontaneously broken, *i.e.*, $m^2 < 0$; and we mainly consider small coupling constants λ, because this slows the dynamics and allows a better study of its different parts.

[a] e-mail: francao@fis.ucm.es

We consider here the evolution of a initial thermal state of temperature T after a flip in the homogeneous external field $\mathcal{J} \to -\mathcal{J}$. We can choose the axes in the N-dimensional internal space such that

$$\mathcal{J} = \begin{cases} (\sqrt{N}J, 0, \dots, 0) & \text{for } t \le 0, \\ (-\sqrt{N}J, 0, \dots, 0) & \text{for } t > 0. \end{cases} \tag{3}$$

For an initial thermal state we have an expectation value parallel to the external field. The following decomposition can be done:

$$\boldsymbol{\Phi}(x) = (\sigma(x), \boldsymbol{\pi}(x)) = \left(\sqrt{N}\phi(t) + \chi(x),\, \boldsymbol{\pi}(x)\right), \tag{4}$$

with $\sqrt{N}\phi(t) = \langle\sigma(x)\rangle$; thus, $\langle\chi(x)\rangle = 0$. While in the remaining $N-1$ directions transversal to the expectation value, $\langle\boldsymbol{\pi}(x)\rangle = 0$.

3 Evolution equations in the large-N limit

As we have one direction parallel to the expectation value and $N-1$ transversal, the fluctuations in the transverse directions dominate in the large-N limit, while those in the longitudinal direction only contribute to the evolution equations as corrections of order $1/N$.

The large-N limit provides explicit evolution equations for the expectation value and the modes of the quantum fluctuations. See refs. [3] and [4] for the detailed expressions. We will discuss here the results obtained with this evolution equations, in particular the effects of the initial temperature. In order to simplify the expression of the results, we introduce the following adimensional variables:

$$\tau \equiv |m|t\,, \quad \eta(\tau) \equiv \sqrt{\tfrac{\lambda}{2}}\,\tfrac{\phi(t)}{|m|}\,, \quad \jmath \equiv \sqrt{\tfrac{\lambda}{2N}}\,\tfrac{\mathcal{J}}{|m|^3}\,, \tag{5}$$

$$g \equiv \tfrac{\lambda}{8\pi^2}\,, \quad \beta \equiv \tfrac{\beta_d}{|m|}\,, \quad g\Sigma(\tau) \equiv \tfrac{\lambda}{2|m|^2}\,\tfrac{\langle \pi^2\rangle(t)}{N}\,. \tag{6}$$

The fact that the initial state is not the ground state but a thermal state increases the initial value of $g\Sigma$. It can be shown that $\Sigma(0)$ is approximately given by

$$\Sigma(0) = \Sigma^{T=0}(0) + \frac{\pi^2}{3}\beta^{-2}\,. \tag{7}$$

$\Sigma^{T=0}(0)$ is the zero-temperature value, that for $g \ll 1$ only depends on j (see ref. [3]). The second term on the right-hand side of eq. (7) is the thermal contribution computed in the hard thermal loop approximation [5]. This expression for $\Sigma(0)$ implies that it is greater for larger initial temperatures.

4 Effective dynamical potential for the expectation value

We can define a potential

$$V_{de;\tau>0}(\eta,\Sigma) \equiv \frac{\eta^4}{4} - \frac{\eta^2}{2} + \frac{1}{2}\eta^2 g\Sigma - \frac{g\Sigma}{2} + \frac{(g\Sigma)^2}{4} + \frac{1}{4} + j\eta\,. \tag{8}$$

that can be interpreted as a dynamical effective potential because the evolution equation for η (in the large-N limit) can be written as [3,4]

$$\ddot{\eta}(\tau) = -\frac{\partial}{\partial\eta}V_{de;\tau>0}(\eta,\Sigma)\,. \tag{9}$$

It must be stressed that $V_{de;\tau>0}$ is an effective potential *only* for η (and *not* for the modes).

5 Equilibrium states for the dynamical potential

The dynamical effective potential for $\tau \le 0$ can be defined as $V_{de;\tau\le 0}(\eta,\Sigma) = V_{de;\tau>0}(\eta,\Sigma) - 2j\eta$. Therefore, the stationary states for the initial dynamics for times $\tau \le 0$ (before the external field has been flipped) are the solutions of $V'_{de;\tau\le 0}(\eta) = 0$ (the prime means η derivative). Thus, they verify

$$\eta^3 + (-1 + g\Sigma)\eta - j = 0\,. \tag{10}$$

For small external fields,

$$j < j_d \equiv 2\sqrt{\frac{(1-g\Sigma)}{27}}\,, \tag{11}$$

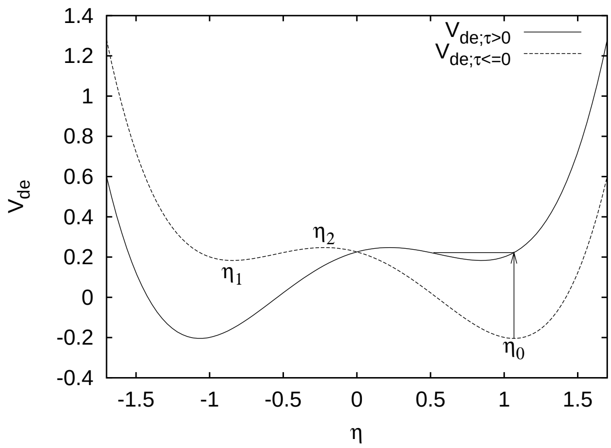

Fig. 1. Dynamical effective potential $V_{de;\tau\le 0}$ and $V_{de;\tau>0}$ as a function of η for $g\Sigma(0) = 0.05$. The value of the external field is $j = 0.2$. The positions of the global minimum η_0, the local minimum η_1, and the local maximum η_2 are shown. There is a potential barrier ($j < j_d$), and the systems gets temporally trapped ($j < j_c$).

we have three roots. There is a global minimum that corresponds to a stable equilibrium state, a local minimum that corresponds to a metastable equilibrium state, and a local maximum (unstable equilibrium) (see fig. 1). On the other hand, for larger external fields ($j > j_d$) there is a single extreme that corresponds to a global minimum (stable equilibrium).

As a larger initial temperature increases $g\Sigma(0)$, it contributes to restoring the symmetry and as consequence makes the value of j_d lower (see eq. (11)).

We consider here the more interesting case $j < j_d$, where a potential barrier is present.

6 Early-time dynamics and dynamical regimes

After the initial flip of the external field sign at $\tau = 0$ the positions of the absolute minimum and the relative minimum of the potential are interchanged, and the state of the system becomes a metastable state (fig. 1). The existence of a potential barrier gives rise to two different dynamical regimes. In the first one the system can directly overcome the barrier, and rapidly reaches the neighborhoods of the global minimum. While in the second regime the system cannot overcome the barrier directly, and it gets temporally trapped close to the metastable state. The external field critical value j_c that separates the two regimes, untrapped $|j| > j_c$, or trapped $|j| < j_c$, is

$$j_c(\beta) = \frac{j_c^{T=0}}{\left[1 - \left(\frac{\beta_c^{J=0}}{\beta}\right)^2\right]^{3/2}} \tag{12}$$

with $j_c^{T=0} = \sqrt{2\,\frac{(13^2+15\sqrt{5})}{19^3}} = 0.243019\ldots$, and $\beta_c^{J=0} = \pi\sqrt{\frac{g}{3}}$. Therefore, we see that if the initial temperature is

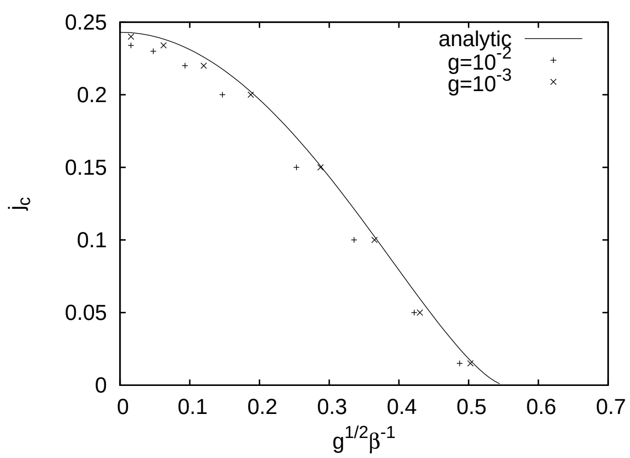

Fig. 2. Critical external field j_c as a function of β^{-1} for $g = 10^{-2}$ and $g = 10^{-3}$ obtained from numerical simulations (symbols) and from the analytical formula in eq. (12) (full line). There are two regions in the (j, β^{-1})-plane corresponding to the two dynamical regimes: for $j < j_c$ the system is temporally trapped in the metastable state; for $j > j_c$ the system rapidly reaches the neighborhood of the global minimum of the potential.

higher enough the system will not be trapped. In addition, eq. (12) states how the trapping can disappear due to a combination of the effects of both the initial temperature and the external field (see fig. 2).

When the trapping is present spinodal instabilities made the modes grow and finally their backreaction allows the system to go close to the stable state. A detailed analysis of the spinodal instability allows to obtain the trapping time, or spinodal time τ_s (see refs. [3,4]). At early times ($\tau < \tau_s$), the effective squared mass oscillates with a negative average, $-\mu^2 \simeq -j$, implying the growth of the modes due to spinodal instability. This implies a quasiexponential growth of $g\Sigma(\tau)$ for $\tau < \tau_s$

$$g\Sigma_s(\tau) \approx \begin{cases} \frac{2}{\beta\mu} g\Sigma_s^{T=0}(\tau) & \text{for } \beta^{-1} \gg \mu\,, \\ g\Sigma_s^{T=0}(\tau) & \text{for } \beta^{-1} \ll \mu\,, \end{cases} \tag{13}$$

with

$$g\Sigma_s^{T=0}(\tau) \approx \frac{g\sqrt{\pi\mu}e^{2\tau\mu}}{8\tau^{3/2}} \tag{14}$$

the value for zero temperature. After a certain time, the spinodal time τ_s, the quantum and thermal effects start to be important in the dynamics, $g\Sigma_s(\tau_s)$ compensates $-\mu^2$ and the exponential growth of the mode functions stops, then the mode functions start to have an oscillatory behavior. Thus, the spinodal time τ_s is defined as

$$g\Sigma_s(\tau_s) = \mu^2\,. \tag{15}$$

A good approximate expression for the spinodal time is

$$\tau_s = \begin{cases} \frac{1}{2\mu}\left[\log\left(\frac{8}{g\sqrt{\pi}}\right) - \log\left(\frac{2}{\beta\mu}\right) + \frac{3}{2}\log(\mu\tau_s)\right] & \text{for } \beta^{-1} \gg \mu \\ \frac{1}{2\mu}\left[\log\left(\frac{8}{g\sqrt{\pi}}\right) + \frac{3}{2}\log(\mu\tau_s)\right] & \text{for } \beta^{-1} \ll \mu \end{cases} \tag{16}$$

For $\beta^{-1} \gg \mu$ we see that a higher initial temperature implies a shorter trapping period.

7 Intermediate-time dynamics

After the early dynamics described in the previous section, the system enters a quasiperiodic regime, both for $j < j_c$ and for $j > j_c$. The system at this intermediate times presents a clear separation between fast variables and slow variables. $\eta(\tau)$, $g\Sigma(\tau)$, and the effective squared mass oscillate fast, while the amplitude of their oscillations slowly decreases. This quasiperiodic regime in the large-N limit evolution equations is present both for zero and nonzero initial temperatures. (See refs. [3,4].)

8 Conclusions

One of the main features of the dynamics is the existence in some cases of a trapping stage in the dynamics that have been characterized. In particular, we have seen that the main influence of the initial temperature is to contribute to the restoration of the symmetry of the potential, and as a consequence a higher initial temperature shortens or even avoids the initial trapping. For intermediate times we have found a quasiperiodic regime. However, how much this dynamics will be modified by the inclusion of next–to–leading-order terms in the large-N approximation [6] is still an open question.

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